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Semi-analytical approach to Higgs production at LEP 2*

Dima Bardin[a b †], Arnd Leike[c ‡] and Tord Riemann[d]

[a]Joint Institute for Nuclear Research, ul. Joliot-Curie 6, RU–141980 Dubna, Moscow Region, Russia

[b]Theoretical Physics Division, CERN, CH–1211 Geneva 23, Switzerland

[c]Lehrstuhl Prof. Fritzsch, Sektion Physik der Ludwig-Maximilians-Universität, Theresienstr. 37, D–80333 München, Germany

[d]DESY–Institut für Hochenergiephysik, Platanenallee 6, D–15738 Zeuthen, Germany

The cross-section for the reaction $e^+e^- \to b\bar{b}\mu^+\mu^-$ is calculated with a semi-analytical integration of the phase space. Compact formulae are obtained for the total cross section and for invariant mass distributions of the $\mu^+\mu^-$ and $b\bar{b}$ pairs. The background diagrams to ZH production yield analytically cumbersome but numerically small contributions. The numerical results are compared with those from a Monte Carlo approach.

1. INTRODUCTION

A main task of LEP 2 will be the investigation of the production of two heavy gauge bosons. As it is well known, these particles are extremely unstable. Immediately after their production, they decay preferably into a four-fermion final state. The same final states are produced by competing background reactions with one or no gauge boson in the intermediate states.

A spectacular event could be the observation of Higgs production at LEP 2 via the Bjorken process [1]:

$$e^+e^- \to ZH. \tag{1}$$

For the Standard Model Higgs mass range of interest at LEP, it is $M_H < 2M_W$ and the Higgs boson decays with a probability of almost 100% into a pair of b-quarks. Therefore, the competing reactions to Higgs production at LEP 2 are four-fermion final states containing a b-quark pair. Monte Carlo calculations of the processes

$$e^+e^- \quad \to \quad Zb\bar{b}, \tag{2}$$

$$e^+e^- \quad \to \quad \mu^+\mu^-b\bar{b} \tag{3}$$

have been performed in [2].

In this contribution, we apply our semi-analytical approach [3] to the calculation of the cross-section of reaction (3). However, our analytical results are also applicable to a full class of similar processes:

$$e^+e^- \to f_1\bar{f}_1f_2\bar{f}_2, \tag{4}$$

with $f_i \neq e, \nu$ and $f_1 \neq f_2$.

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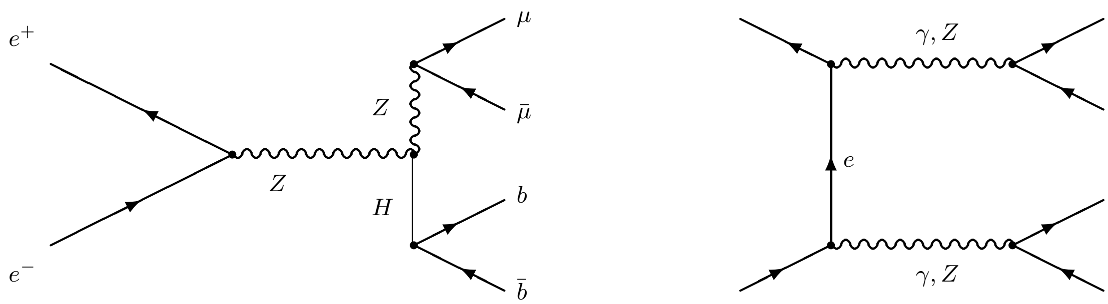

Figure 1. The basic diagram for off-shell ZH production (a) and the `crab` type background (b).

2. AMPLITUDES AND INTERFERENCES

The process (3) is described by seven generic diagrams:

(i) The Higgs signal diagram, figure 1a;

(ii) t and u channel exchange background diagrams of the `crab` type, figure 1b;

(iii) four non-resonating background diagrams of the `(rein)deer` type, figure 2.

Every of the generic diagrams in (ii) and (iii) represents four Feynman diagrams with the virtual neutral gauge bosons being photons or Z bosons. Thus, the whole process is described by $1+4\cdot 2+4\cdot 4=25$ Feynman diagrams. The complexity of the problem arises from the properties of the seven types of generic diagrams. Here we only mention that the interferences of the Higgs diagram with all the others vanish after an integration over the angles characteristic of the b-quark pair – with one exception: the two `deer` diagrams with the b-quarks being attached to the first intermediate γ, Z in figure 2 ($f_1 = b$). All the other interferences with the signal yield the same trace over the b-quark line:

$$Tr\left[(\not{p}_b + m)(\not{p}_{\bar{b}} - m)\gamma^\alpha\right] = 4m(p_{\bar{b}} - p_b)^\alpha. \quad (5)$$

After an integration over the phase space of the b-quarks:

$$\begin{aligned} \sigma^{\rm int} &= \int {\rm d}\Omega_b (p_{\bar{b}} - p_b)^\alpha T_\alpha(k_1, k_2, p_{\mu^+}, p_{\mu^-}) \\ &= 0. \end{aligned} \quad (6)$$

Here it is essential that T_α is independent of the momenta $p_{\bar{b}}$ and p_b. So, the integrand changes sign when interchanging $p_{\bar{b}}$ and p_b but $d\Omega_b$ does not. As a result, the integral vanishes after the integration over the b-quark angles.

Thus, the Higgs signal adds up incoherently with the `crab` contributions and a large fraction of the other background if not b-quark asymmetries are studied.

In addition, we would like to mention that all the interferences of the Higgs signal with background diagrams are suppressed with respect to the squared Higgs diagram due to the narrow width of the Higgs boson. A rough estimate may be obtained as follows. Be $\chi_B(s)$ a boson propagator,

$$\chi_B(s) = \frac{s}{s - M_B^2 + i\Gamma_B M_B}. \quad (7)$$

If there is a chance at all to find a Higgs boson at LEP 2, then it is not unrealistic to assume both Higgs boson and the Z to be nearly on their mass shells. Then, the ratio of the propagators may be estimated to be roughly as follows:

$$\frac{\chi_Z}{\chi_H} \approx \frac{\Gamma_Z}{\Gamma_H}, \qquad \frac{\chi_\gamma}{\chi_H} \approx \frac{M_H}{\Gamma_H}. \quad (8)$$

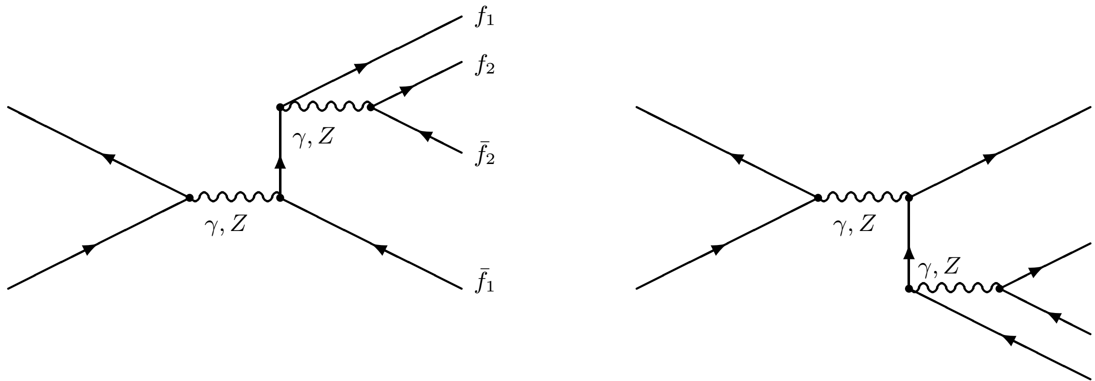

Figure 2. Background contributions to off-shell ZH production: up and down `reindeers`; $f_i = \mu, b$.

Below the threshold of the decay $H \to W^+W^-$, the off-shell width of the H boson is

$$\Gamma_H(s) = \frac{G_\mu}{4\pi\sqrt{2}}\sqrt{s}\sum_f m_f^2 N_c(f), \tag{9}$$

and that of the Z is

$$\Gamma_Z(s) = \frac{G_\mu\, M_Z^2}{24\pi\sqrt{2}}\sqrt{s}\sum_f (v_f^2 + a_f^2) N_c(f), \tag{10}$$

where $N_c(f) = 1$ (3) for leptons (quarks). We use the normalization $a_f = 1$.

For $M_H < 2M_W$ the Higgs width is of the order of a few MeV and the non-vanishing interferences of background with the Higgs signal are highly suppressed at LEP 2.

3. PHASE SPACE AND CROSS SECTIONS

We parametrize the eight-dimensional phase space of four final state particles as follows:

$$d\Omega = \prod_{i=1}^{4}\frac{d^3p_i}{2p_i^0}\delta^4(k_1 + k_2 - \sum_{i=1}^{4} p_i) \tag{11}$$

$$= 2\pi\frac{\sqrt{\lambda(s, s_H, s_Z)}}{8s}\,\frac{\sqrt{\lambda(s_H, m_b^2, m_b^2)}}{8s_H}$$

$$\times \frac{\sqrt{\lambda(s_Z, m_\mu^2, m_\mu^2)}}{8s_Z} ds_H ds_Z d\cos\theta d\Omega_H d\Omega_Z,$$

where we already integrated over the rotation angle around the beam axis. The k_1 and k_2 are the four-momenta of the initial electron and positron and p_i those of the final state particles. The invariants s, s_H, and s_Z are

$$\begin{aligned} s &= (k_1 + k_2)^2, \\ s_H &= (p_1 + p_2)^2, \qquad s_Z = (p_3 + p_4)^2, \end{aligned} \tag{12}$$

and θ is the angle between the vectors $(\vec{p_1} + \vec{p_2})$ and $\vec{k}_1$. The spherical angles of the momenta $\vec{p_1}$ and $\vec{p_2}$ ($\vec{p_3}$ and $\vec{p_4}$) in their rest frames are in Ω_H (Ω_Z): $d\Omega_i = d\cos\theta_i d\phi_i$. The kinematical ranges of the integration variables are:

$$\begin{aligned} (2m_b)^2 &\le s_H \le (\sqrt{s} - 2m_\mu)^2, \\ (2m_\mu)^2 &\le s_Z \le (\sqrt{s} - \sqrt{s_H})^2, \\ -1 &\le \cos\theta,\ \cos\theta_H,\ \cos\theta_Z \le 1, \\ 0 &\le \phi_H, \phi_Z \le 2\pi. \end{aligned} \tag{13}$$

We integrated analytically over all the angular variables, leaving the integrations over s_H and s_Z to be performed numerically.

The cross section is:

$$\sigma(s) = \int_{\bar{s}_H}^{s} ds_H \rho(s_H)\int_{\bar{s}_Z}^{(\sqrt{s}-\sqrt{s_H})^2} ds_Z \times \rho(s_Z)\sigma_0(s, s_H, s_Z), \tag{14}$$

with

$$\rho(s) = \frac{1}{\pi}\frac{\sqrt{s}\,\Gamma(s)}{|s - M^2 + i\sqrt{s}\,\Gamma(s)|^2}\cdot \mathrm{BR}. \tag{15}$$

The M and Γ are mass and width of the resonating off shell particles and BR is the branching ratio of its decay to the observed final state fermion

pair. The $\rho(s)$ has the property $\lim_{\Gamma\to 0}\rho(s) \to \delta(s-M^2)$BR. The lower integration bounds $\bar{s}_H$ and $\bar{s}_Z$ cut on the invariant masses of the b-quark and muon pairs.

The functions $\sigma_0(s, s_H, s_Z)$ in (14) are the result of a fivefold analytical integration. The result has the following structure:

$$\sigma_0(s, s_H, s_Z) = \sigma_0^H + \sigma_0^{H,b-\texttt{deers}} + \sigma_0^{\texttt{crab}} + \sigma_0^{\texttt{deers}} + \sigma_0^{\texttt{crab},\texttt{deers}}. \quad (16)$$

Here, the superscripts denote the different interferences of the generic diagrams. The numerically largest contributions are

$$\sigma_0^H(s; s_1, s_2) = \frac{\left(G_\mu M_Z^2\right)^2}{96\pi s}\frac{M_Z^2}{s}\left(v_e^2 + a_e^2\right) \times \left|\frac{s}{s - M_Z^2 + iM_Z\Gamma_Z(s)}\right|^2 \mathcal{G}_4^{\mathrm{Bj}}(s; s_1, s_2) \quad (17)$$

with the kinematical function

$$\mathcal{G}_4^{\mathrm{Bj}}(s; s_1, s_2) = \frac{\lambda^{1/2}}{s^2 s_2}\left(\lambda + 12 s s_2\right) \quad (18)$$

and [3]

$$\sigma_0^{\texttt{crab}}(s; s_1, s_2) = \left[\frac{\left(G_\mu M_Z^2\right)^2}{64\pi s}\left(v_e^4 + 6v_e^2 a_e^2 + a_e^4\right) + \ldots\right]\mathcal{G}_4^{\mathrm{t+u}}(s, s_1, s_2) \quad (19)$$

with

$$\mathcal{G}_4^{\mathrm{t+u}}(s; s_1, s_2) = \frac{\lambda^{1/2}}{s}\left[\frac{s^2 + (s_1 + s_2)^2}{s - s_1 - s_2}\mathcal{L}_4 - 2\right].$$

The dots in (19) indicate the contributions with intermediate photons. Further, the following definitions are used:

$$\lambda \equiv \lambda(s; s_1, s_2) = s^2 + s_1^2 + s_2^2 - 2ss_1 - 2s_1 s_2 - 2s_2 s \quad (20)$$

and

$$\mathcal{L}_4(s; s_1, s_2) = \frac{1}{\sqrt{\lambda}}\ln\frac{s - s_1 - s_2 + \sqrt{\lambda}}{s - s_1 - s_2 - \sqrt{\lambda}}. \quad (21)$$

The results for the `deer` diagrams and their interferences are rather lengthy and will be published elsewhere[4].

[4]The `crab` and `deer` cross sections alone have been calculated in a Monte Carlo approach in [4].

The above results may be used for the calculation not only of total cross sections but also of distributions $\mathrm{d}\sigma/\mathrm{d}s_H$, $\mathrm{d}\sigma/\mathrm{d}s_Z$, and $\mathrm{d}\sigma/\mathrm{d}\cos\theta$ which are of vital importance for a Higgs search [5]. Further, b-quark and muon pairs of low invariant mass may be cut in order to avoid non-perturbative effects due to bound states and large background due to photon exchange.

The equations (14) and (17)–(19) contain the numerically important contributions to the reaction $e^+e^- \to b\bar{b}\mu^+\mu^-$ at the Born level. For applications to data, the inclusion of the universal initial state QED corrections is important [6]:

$$\sigma_T = \int_{\bar{s}}^{s}\frac{ds'}{s}\,\sigma(s')\,\rho(s'/s), \quad (22)$$

with $\bar{s} \geq (2m_\mu + 2m_b)^2$. The accuracy then should be quite sufficient for a search experiment although certain QED and weak corrections are left out [3, 7, 8].

4. RESULTS

The total cross section $\sigma(s)$ is shown in figure 3 for various Higgs masses. Cuts on the invariant masses have been applied: $\sqrt{s_H} = E_{b\bar{b}} \geq 12$ GeV, $\sqrt{s_Z} = E_{\mu^+\mu^-} \geq 12$ GeV. We found agreement with results from a Monte Carlo calculation [2] within the errors of the latter. With an integrated luminosity of $500\,fb^{-}1$ one may expect between 10 and 20 $b\bar{b}\mu^+\mu^-$ events.

The b-quark pairs from Higgs decay have a δ-function like peak in the energy distribution, see figure 4. This is due to the small Higgs width which may be estimated in the Standard Model to about 4, 5, 6 MeV for M_H = 80, 100 and 140 GeV. This way, b-quark pairs from Higgs decay may easily be distinguished from background even if $M_H \approx M_Z$. Although a Higgs with a mass of 120 GeV could give a strong peak in the energy distribution of the b-pairs, it would be too heavy to be detected at LEP 2 because it gives less then 1 event for the considered luminosity even if $\sqrt{s}$ = 200 GeV would be realized; see figure 5. The maximal mass for which a Higgs boson detection seems feasible at LEP 2 is $M_H \approx \sqrt{s} - 100\,GeV$ [5].

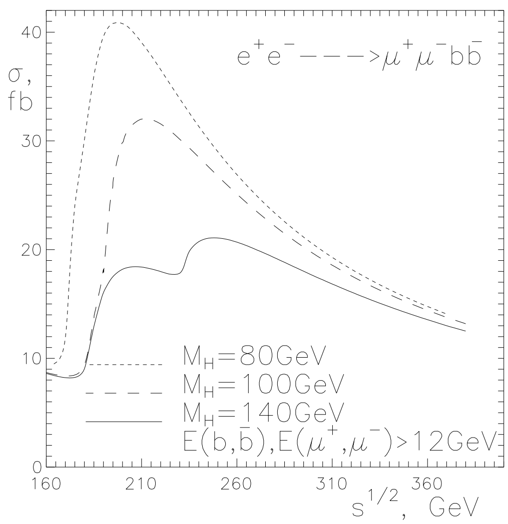

Figure 3. The total cross section $\sigma(e^+e^- \to b\bar{b}\mu^+\mu^-)$ as function of $s^{1/2}$ for various Higgs masses.

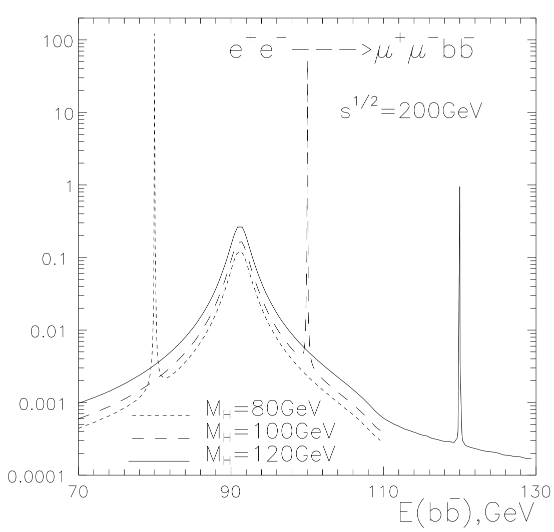

Figure 4. The distribution $\mathrm{d}\sigma/\mathrm{d}E_{b\bar{b}} \cdot E_{beam}/\sigma$ for different Higgs masses at $s^{1/2} = 200\,\mathrm{GeV}$. A cut $E_{\mu^+\mu^-} \geq 12$ GeV has been applied.

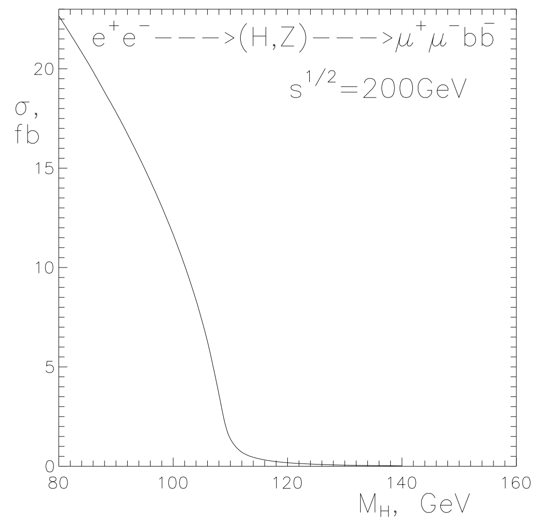

Figure 5. The total cross section $\sigma(e^+e^- \to (H, Z) \to b\bar{b}\mu^+\mu^-)$ as a function of the Higgs mass for $s^{1/2} = 200\,\mathrm{GeV}$.

To summarize, we performed the first complete semi-analytical calculation of the off-shell Bjorken process, $e^+e^- \to b\bar{b}\mu^+\mu^-$. The interferences between the Higgs signal and the resonating `crab` background are zero after integration over the angles of the b-quark pair. The rest of the background may be neglected in a search experiment. The analytical results are applicable also for reactions of the type $e^+e^- \to f_1\bar{f}_1 f_2\bar{f}_2$, where $f_1 \neq f_2$ and $f_1, f_2 \neq e, \nu_e$. This opens the possibility for a further study of e.g. the higher order fermion pair corrections to the Z line shape [9].

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